Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 40 minutes

Paper reference **8FM0/01**

Further Mathematics

Advanced Subsidiary
PAPER 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P68730A

Q:1/1/1/1/1/

P 6 8 7 3 0 A 0 1 3 2

Pearson

1.

$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 7 & 2 \\ -5 & 8 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 6 & 5 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}$$

Given that $\mathbf{I}$ is the 3×3 identity matrix,

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) (i) show that there is an integer k for which

$$\mathbf{AB} - 3\mathbf{C} + k\mathbf{I} = \mathbf{0}$$

stating the value of k

(ii) explain why there can be no constant m such that

$$\mathbf{BA} - 3\mathbf{C} + m\mathbf{I} = \mathbf{0}$$

(4)

(b) (i) Show how the matrix $\mathbf{C}$ can be used to solve the simultaneous equations

$$\begin{aligned} -5x + 2y + z &= -14 \\ 4x + 3y + 8z &= 3 \\ -6x + 11y + 2z &= 7 \end{aligned}$$

(ii) Hence use your calculator to solve these equations.

(3)

(a)(i) $AB = \begin{bmatrix} 4 & -1 \\ 7 & 2 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 \\ -1 & 6 & 5 \end{bmatrix}$

$(3 \times \cancel{2})(\cancel{2} \times 3)$

result will be 3×3

$$AB = \begin{bmatrix} 4 \times 2 + -1 \times -1 & 4 \times 3 + -1 \times 6 & 4 \times 2 + -1 \times 5 \\ 7 \times 2 + 2 \times -1 & 7 \times 3 + 2 \times 6 & 7 \times 2 + 2 \times 5 \\ -5 \times 2 + 8 \times -1 & -5 \times 3 + 8 \times 6 & -5 \times 2 + 8 \times 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 9 & 6 & 3 \\ 12 & 33 & 24 \\ -18 & 33 & 30 \end{bmatrix}$$ 1

$$3 \times C = \begin{bmatrix} -15 & 6 & 3 \\ 12 & 9 & 24 \\ -18 & 33 & 6 \end{bmatrix}$$

Question 1 continued

$$AB - 3C = \begin{pmatrix} 9 & 6 & 3 \\ 12 & 33 & 24 \\ -18 & 33 & 30 \end{pmatrix} - \begin{pmatrix} -15 & 6 & 3 \\ 12 & 9 & 24 \\ -18 & 33 & 6 \end{pmatrix}$$

$$AB - 3C = \begin{pmatrix} 9--15 & 6-6 & 3-3 \\ 12-12 & 33-9 & 24-24 \\ -18--18 & 33-33 & 30-6 \end{pmatrix} = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix} \quad 1$$

$$AB-3C + kI = 0 \Rightarrow AB-3C = -kI$$

$$-k\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix} \quad \therefore k = -24 \quad 1$$

(ii) B is 2×3 and A is 3×2 so BA will be a 2×2 matrix. 1 3C is a 3×3 matrix and you cannot subtract matrices with different dimensions. 1

(b)(i)
$$\begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix}$$

← if you multiply these out it will give the three equations from the question

eg. $-5\times x + 2\times y + 1\times z = -14$

$-5x + 2y + z = -14$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} \quad 1$$

$A^{-1} = \frac{1}{\det A} \times \text{transpose}(A)$

(do this on a calculator)

(ii)
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{360}\begin{pmatrix} -82 & 7 & 13 \\ -56 & -4 & 44 \\ 62 & 43 & -23 \end{pmatrix}\begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} \quad 1$$

PMT

Question 1 continued

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{41}{180} & \frac{7}{360} & \frac{13}{360} \\ -\frac{7}{45} & -\frac{1}{90} & \frac{11}{90} \\ \frac{31}{180} & \frac{43}{360} & -\frac{23}{360} \end{pmatrix} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} \quad 1$$

Use a calculator!

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7/2 \\ 3 \\ -5/2 \end{pmatrix} \quad \therefore \; x = \frac{7}{2}, \quad y = 3, \quad z = -\frac{5}{2} \quad 1$$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 7 marks)

2. (a) Express the complex number $w = 4\sqrt{3} - 4\text{i}$ in the form $r(\cos\theta + \text{i}\sin\theta)$ where $r > 0$ and $-\pi < \theta \leqslant \pi$

(4)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by $\arg(z + 10\text{i}) = \dfrac{\pi}{3}$

(3)

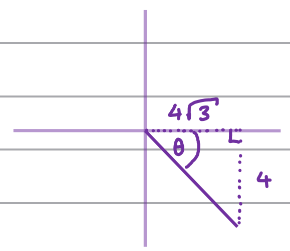
(c) Hence determine the minimum distance of w from the locus $\arg(z + 10\text{i}) = \dfrac{\pi}{3}$

(3)

(a) $|w| = \sqrt{(4\sqrt{3})^2 + (-4)^2} = \sqrt{48 + 16} = \sqrt{64} = 8$ 1

$\arg(w) = \tan^{-1}\left(\frac{\pm 4}{4\sqrt{3}}\right)$ 1 $= -\frac{\pi}{6}$ 1

$w = 8\left(\cos\left(-\frac{\pi}{6}\right) + \text{i}\sin\left(-\frac{\pi}{6}\right)\right)$ 1



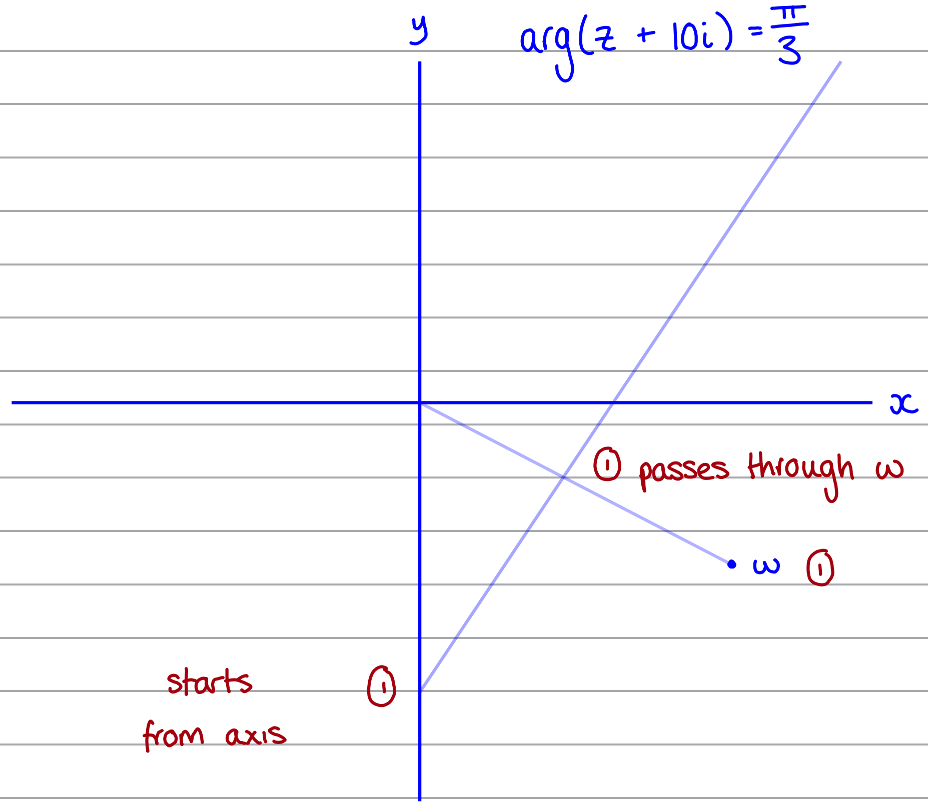
$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{4}{4\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)$

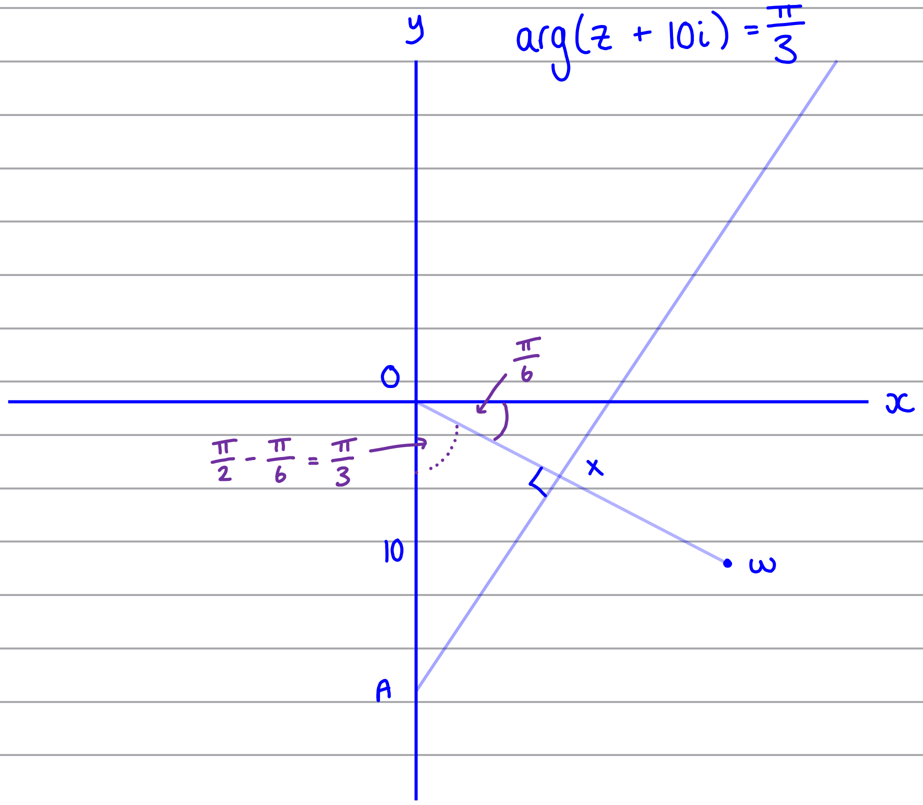
(b)(i)

(ii)



DO NOT WRITE IN THIS AREA

Question 2 continued



OXA is a right angle.

$\cos\theta = \frac{A}{H}$

$\cos\frac{\pi}{3} = \frac{OX}{10}$

from $|w|$

$OX = 10\cos\frac{\pi}{3} = 5$ (1)

$WX = WO - OX = 8 - 5 = 3$ (1)

$\therefore$ min distance is 3. (1)

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. *With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the y-axis is represented by the matrix*

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

The point P has coordinates (8, 3, 2)

The point Q is the image of P under the transformation reflection in the plane $y = 0$

(a) Write down the coordinates of Q

(1)

The point R is the image of P under the transformation rotation through 120° anticlockwise about the y-axis, with respect to the **right-hand rule**.

(b) Determine the exact coordinates of R

(2)

(c) Hence find $|\overrightarrow{PR}|$ giving your answer as a simplified surd.

(2)

(d) Show that $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular.

(1)

(e) Hence determine the exact area of triangle PQR, giving your answer as a surd in simplest form.

(2)

(a) $Q = (8, -3, 2)$ ① y-coordinate becomes negative

(b) $\theta = 120°$

$$R = \begin{bmatrix} \cos 120 & 0 & \sin 120 \\ 0 & 1 & 0 \\ -\sin 120 & 0 & \cos 120 \end{bmatrix} \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix}$$ ① ← transformation before co-ordinate!

$$R = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix}$$ ①

P 6 8 7 3 0 A 0 1 0 3 2

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

$$R = \begin{pmatrix} -\frac{1}{2}\times 8 + \frac{\sqrt{3}}{2}\times 2 \\ 1\times 3 \\ -\frac{\sqrt{3}}{2}\times 8 + -\frac{1}{2}\times 2 \end{pmatrix}$$

$$R = \begin{pmatrix} -4+\sqrt{3} \\ 3 \\ -4\sqrt{3}-1 \end{pmatrix} \qquad \therefore\ R \text{ is } (-4+\sqrt{3},\ 3,\ -4\sqrt{3}-1) \ (1)$$

(c) Distance PR $= \sqrt{(8-(-4+\sqrt{3}))^2 + (3-3)^2 + (2-(-4\sqrt{3}-1))^2}$ (1)

$$|\overrightarrow{PR}| = \sqrt{204} = 2\sqrt{51} \ (1)$$

(d) $$\overrightarrow{PR}\cdot\overrightarrow{PQ} = \begin{pmatrix} -12+\sqrt{3} \\ 0 \\ -3-4\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} = 0 \ (1)$$

$\therefore$ $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular.

(e) area $= \frac{1}{2}\times PQ \times PR$ (1) $\leftarrow$ because PR and PQ are perpendicular so they form a triangle.

$$= \frac{1}{2}\times 6 \times \sqrt{204}$$

$$= 6\sqrt{51} \ (1)$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

PMT

4. The roots of the quartic equation

$$3x^4 + 5x^3 - 7x + 6 = 0$$

a b (c) d e

are α, β, γ and δ

Making your method clear and without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

(3)

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma} + \dfrac{2}{\delta}$

(3)

(iii) $(3 - \alpha)(3 - \beta)(3 - \gamma)(3 - \delta)$

(3)

(i) $(\alpha+\beta+\gamma+\delta)^2 = \alpha^2 + \alpha\beta + \alpha\gamma + \alpha\delta + \beta^2 + \alpha\beta + \beta\gamma + \beta\delta + \gamma^2 + \alpha\gamma + \beta\gamma + \gamma\delta + \delta^2 + \alpha\delta + \beta\delta + \gamma\delta$

$= \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$

1

$\therefore \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$

$\alpha + \beta + \gamma + \delta = -\dfrac{b}{a} = -\dfrac{5}{3}$

$(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta) = \dfrac{c}{a} = \dfrac{0}{3} = 0$ 1

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \left(-\dfrac{5}{3}\right)^2 - 2 \times 0 = \dfrac{25}{9}$ 1

(ii) let $x = \dfrac{2}{\omega}$ 1 ← transform the equation to match the new roots

eg $\alpha \rightarrow \dfrac{2}{\alpha}$

then $3\left(\dfrac{16}{\omega^4}\right) + 5\left(\dfrac{8}{\omega^3}\right) - 7\left(\dfrac{2}{\omega}\right) + 6 = 0$

$\dfrac{48}{\omega^4} + \dfrac{40}{\omega^3} - \dfrac{14}{\omega} + 6 = 0$ $\times\omega^4$

$6\omega^4 - 14\omega^3 + 40\omega + 48 = 0$ 1

P 6 8 7 3 0 A 0 1 4 3 2

DO NOT WRITE IN THIS AREA

Question 4 continued

$$\alpha+\beta+\gamma+\delta = -\frac{b}{a}$$

$$\therefore \frac{2}{\alpha}+\frac{2}{\beta}+\frac{2}{\gamma}+\frac{2}{\delta} = -\left(\frac{-14}{6}\right) = \frac{14}{6} = \frac{7}{3} \quad 1$$

(iii) let $x = 3-w$

then $3(3-w)^4 + 5(3-w)^3 - 7(3-w) + 6 = 0 \quad 1$

$$3(81-108w+54w^2-12w^3+w^4)+5(27-27w+9w^2-w^3)$$
$$-21+7w+6 = 0$$

$$\underset{e}{363} - \underset{d}{452w} + \underset{c}{207w^2} - \underset{b}{41w^3} + \underset{a}{3w^4} = 0 \quad 1$$

$$\alpha\beta\gamma\delta = \frac{e}{a} = \frac{363}{3} = 121 \quad 1$$

from this equation

Turn over

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

PMT

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

5. (a) Use the standard summation formulae to show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n}(3r^2 - 17r - 25) = n(n^2 - An - B)$$

where A and B are integers to be determined.

(4)

(b) Explain why, for $k \in \mathbb{N}$,

$$\sum_{r=1}^{3k} r\tan(60r)^\circ = -k\sqrt{3}$$

(2)

Using the results from part (a) and part (b) and showing all your working,

(c) determine any value of n that satisfies

$$\sum_{r=5}^{n}(3r^2 - 17r - 25) = 15\left[\sum_{r=6}^{3n} r\tan(60r)^\circ\right]^2$$

(6)

(a) $\sum_{r=1}^{n}(3r^2-17r-25) = 3\left[\frac{1}{6}n(n+1)(2n+1)\right] - 17\left[\frac{1}{2}n(n+1)\right] - 25n$ 1

$= \frac{1}{2}n(2n^2+3n+1) - \frac{17}{2}n(n+1) - 25n$ 1

$= n\left[\frac{1}{2}(2n^2+3n+1) - \frac{17}{2}(n+1) - 25\right]$

$= \frac{n}{2}\left[(2n^2+3n+1) - 17(n+1) - 50\right]$

$= \frac{n}{2}(2n^2+3n+1-17n-17-50)$ 1

$= \frac{n}{2}(2n^2-14n-66)$

$= n(n^2-7n-33)$

$\therefore A = 7, \quad B = 33$ 1

P 6 8 7 3 0 A 0 1 8 3 2

DO NOT WRITE IN THIS AREA

Question 5 continued

(b) $\sum_{r=1}^{3k} r\tan(60r)^\circ = 1\times\tan(60\times1) + 2\times\tan(60\times2) + 3\times\tan(60\times3) + 4\times\tan(60\times4) + 5\times\tan(60\times5) + 6\times\tan(60\times6) + \ldots$

$= \tan 60 + 2\tan 120 + 3\tan 180 + 4\tan 240 + 5\tan 300 + 6\tan 360 + \ldots$

$= (\sqrt{3} - 2\sqrt{3} + 0) + (4\sqrt{3} - 5\sqrt{3} + 0) + \ldots$ (1)

$\tan\theta$ repeats every 180° so $\tan(60r)$ repeats every 3 terms.

Each group results in $-\sqrt{3}$ as a sum, so with k groups the sum is $-k\sqrt{3}$. (1)

(c) $\sum_{r=5}^{n}(3r^2 - 17r - 25) = \sum_{r=1}^{n}(3r^2 - 17r - 25) - \sum_{r=1}^{4}(3r^2 - 17r - 25)$ (1)

$= n(n^2 - 7n - 33) - 4(4^2 - 7(4) - 33)$

$= n(n^2 - 7n - 33) + 180$ (1)

$\sum_{r=6}^{3n} r\tan(60r)^\circ = -n\sqrt{3} - (-2\sqrt{3})$ (1)

$n(n^2 - 7n - 33) + 180 = 15(-n\sqrt{3} + 2\sqrt{3})^2$

$n^3 - 7n^2 - 33n + 180 = 15(3n^2 - 12n + 12)$

$n^3 - 52n^2 + 147n = 0$ (1)

$n = 49, \quad n = 3, \quad n = 0$ (1)

$n > 5$ for all sums to be valid, so $n = 49$ (1)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6. The surface of a horizontal tennis court is modelled as part of a horizontal plane, with the origin on the ground at the centre of the court, and

- **i** and **j** are unit vectors directed across the width and length of the court respectively
- **k** is a unit vector directed vertically upwards
- units are metres

After being hit, a tennis ball, modelled as a particle, moves along the path with equation

$$\mathbf{r} = \left(-4.1 + 9\lambda - 2.3\lambda^2\right)\mathbf{i} + \left(-10.25 + 15\lambda\right)\mathbf{j} + \left(0.84 + 0.8\lambda - \lambda^2\right)\mathbf{k}$$

where λ is a scalar parameter with $\lambda \geqslant 0$

Assuming that the tennis ball continues on this path until it hits the ground,

(a) find the value of λ at the point where the ball hits the ground.

(2)

The direction in which the tennis ball is moving at a general point on its path is given by

$$\left(9 - 4.6\lambda\right)\mathbf{i} + 15\mathbf{j} + \left(0.8 - 2\lambda\right)\mathbf{k}$$

(b) Write down the direction in which the tennis ball is moving as it hits the ground.

(1)

(c) Hence find the acute angle at which the tennis ball hits the ground, giving your answer in degrees to one decimal place.

(4)

The net of the tennis court lies in the plane $\mathbf{r}.\mathbf{j} = 0$

(d) Find the position of the tennis ball at the point where it is in the same plane as the net.

(3)

The maximum height above the court of the top of the net is 0.9 m.

Modelling the top of the net as a horizontal straight line,

(e) state whether the tennis ball will pass over the net according to the model, giving a reason for your answer.

(1)

With reference to the model,

(f) decide whether the tennis ball will actually pass over the net, giving a reason for your answer.

(2)

Question 6 continued

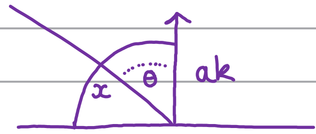
(a) $0.84 + 0.8\lambda - \lambda^2 = 0$ 1 because k component must be 0 on the ground

$\lambda = \frac{7}{5}$, $\lambda = -\frac{3}{5}$ but $\lambda \geq 0$ so $\lambda = \frac{7}{5}$ 1

(b) Direction $= \left(9 - 4.6\left(\frac{7}{5}\right)\right)i + 15j + \left(0.8 - 2\left(\frac{7}{5}\right)\right)k$

$= 2.56i + 15j - 2k$ 1

(c) Direction perpendicular to the ground is ak



$\cos\theta = \dfrac{ak \cdot (2.56i + 15j - 2k)}{a \times |2.56i + 15j - 2k|}$

Angle at impact (x) is $90° - \theta$, so we can use formula to find θ

$\cos\theta = \dfrac{\begin{pmatrix}0\\0\\a\end{pmatrix} \cdot \begin{pmatrix}2.56\\15\\-2\end{pmatrix}}{15.35a}$ 1

$\cos\theta = \dfrac{a \cdot b}{|a||b|}$

$\cos\theta = \dfrac{-2a}{15.35a}$

$\cos\theta = -0.130$ 1 $\Rightarrow \theta = \cos^{-1}(-0.130)$

$x = 90 - \cos^{-1}(-0.130)$ 1

$x = -7.48$

So tennis ball hits the ground at an angle of $7.5°$. 1

P68730A02332

Question 6 continued

(d) In the same plane as net when $r \cdot j = 0$.

$$\begin{pmatrix} -4.1 + 9\lambda - 2.3\lambda^2 \\ -10.25 + 15\lambda \\ 0.84 + 0.8\lambda - \lambda^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -10.25 + 15\lambda \quad 1$$

$$15\lambda = 10.25$$

$$\lambda = \frac{41}{60}$$

$$\left(-4.1 + 9\left(\frac{41}{60}\right) - 2.3\left(\frac{41}{60}\right)^2\right)i + 0j + \left(0.84 + 0.8\left(\frac{41}{60}\right) - \left(\frac{41}{60}\right)^2\right) \quad 1$$

$$= 0.976i + 0.920k \quad 1$$

(e) Modelling the net as a line, it has a height of 0.9m
The k component of the ball is 0.92m, so as
0.92 > 0.9, the ball will pass over the net according to
the model. 1

(k component is vertical height of the ball)

(f) A ball is not a particle 1 - it has a diameter, therefore it
will hit the net and not pass over 1

OR:
↪ as above, but momentum will carry the ball over as it is
mostly above the net.

Question 6 continued

(Total for Question 6 is 13 marks)

7. Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}^n = \begin{pmatrix} 1-6n & 9n \\ -4n & 1+6n \end{pmatrix}$$

(6)

When $n=1$:

$$\begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}^1 = \begin{bmatrix} 1-6(1) & 9(1) \\ -4(1) & 1+6(1) \end{bmatrix}$$

$$\begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}$$

$\therefore$ True for $n=1$. 1 Assume true for all $n=k$, such that :

$$\begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}^k = \begin{bmatrix} 1-6k & 9k \\ -4k & 1+6k \end{bmatrix} \quad 1$$

Consider $n=k+1$

$$\begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}^{k+1} = \begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}^k \times \begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}$$

from previous assumption

$$= \begin{bmatrix} 1-6k & 9k \\ -4k & 1+6k \end{bmatrix} \times \begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} -5(1-6k)-4(9k) & 9(1-6k)+7(9k) \\ -5(-4k)-4(1+6k) & 9(-4k)+7(1+6k) \end{bmatrix}$$

$$= \begin{bmatrix} -5+30k-36k & 9-54k+63k \\ 20k-4-24k & -36k+7+42k \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} -5-6k & 9+9k \\ -4-4k & 7+6k \end{bmatrix} \quad 1$$

DO NOT WRITE IN THIS AREA

Question 7 continued

$$= \begin{bmatrix} 1-6(k+1) & 9(k+1) \\ -4(k+1) & 1+6(k+1) \end{bmatrix}$$

$\therefore$ True for $n=k+1$.

Since the result is true for $n=1$, and if true for $n=k$ then is true for $n=k+1$, thus by mathematical induction the result holds for all $n \in \mathbb{N}$. 1

(Total for Question 7 is 6 marks)

8.

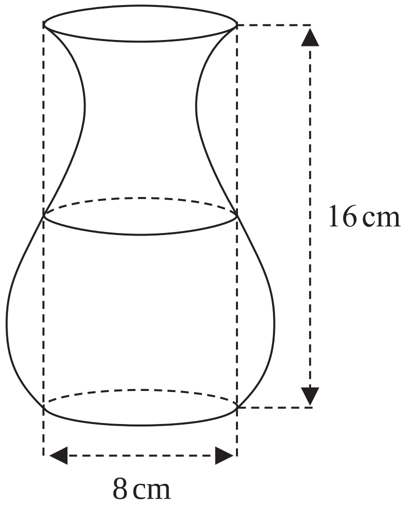


Figure 1

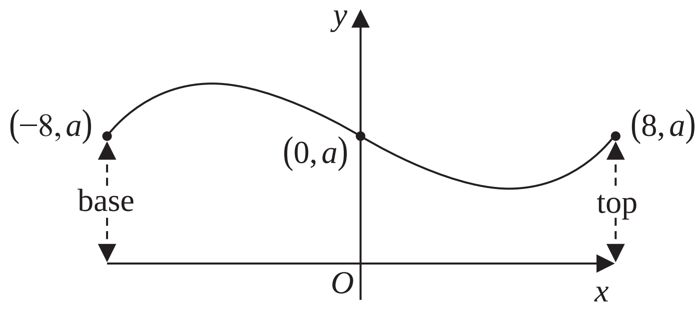


Figure 2

Figure 1 shows a sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross-section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Figure 2, through 360° about the x-axis.

(a) State the value of a that should be used when setting up the model.

(1)

Two possible equations are suggested for the curve in the model.

Model A $\quad y = a - 2\sin\left(\dfrac{45}{2}x\right)^{\circ}$

Model B $\quad y = a + \dfrac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7)

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1)

(d) Using algebraic integration, find the volume for the vase predicted by Model B. You must make your method clear.

(5)

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

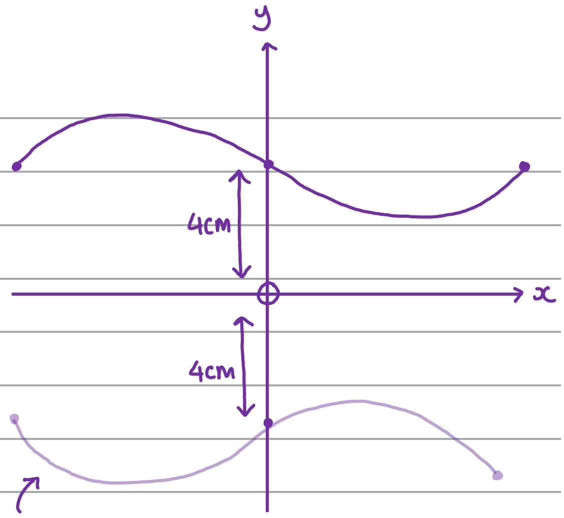
(1)

DO NOT WRITE IN THIS AREA

Question 8 continued

(a) $a = 4$ (1)

diameter of the cross-section is 8cm, so to make the model central on the axis, $a = 8 \div 2 = 4$



(b) Model A: (i) widest point is 4cm from base because sin is symmetrical within half a period

(ii) $y = 4 - 2\sin\left(\frac{45}{2}x - 4\right)$

$y = 6$

$\therefore$ diameter $= 2 \times 6 = 12$cm (1)

Model B: (i) $y = 4 + \dfrac{x^3 - 64x}{100}$ $\rightarrow y = 4 + \frac{1}{100}x^3 - \frac{64}{100}x$

$\frac{dy}{dx} = \dfrac{3x^2 - 64}{100}$ (1) $\leftarrow \frac{dy}{dx} = \frac{3}{100}x^2 - \frac{64}{100}$

$0 = \dfrac{3x^2 - 64}{100}$ (gradient is 0 at turning point)

$0 = 3x^2 - 64$

$64 = 3x^2$

$\pm\sqrt{\dfrac{64}{3}} = x$ (1)

$\therefore$ widest point is $8 - \sqrt{\frac{64}{3}} = 3.38$cm from base. (1)

Question 8 continued

(ii) At widest point, $x = -8 + 3.38 = -4.62$

$$y = 4 + \frac{(-4.62)^3 - 64(-4.62)}{100} \quad (1)$$

$$y = 5.97$$

$$\therefore \text{diameter} = 2 \times 5.97 = 11.94 \text{ cm} \quad (1)$$

(c) Both models gave diameter close to 12cm.
Model B gave a distance of widest point from base closer to 3 than model A, therefore B is more appropriate. (1)

(d) $V_B = \pi \int_{-8}^{8} y^2 \, dx$

$$= \pi \int_{-8}^{8} \left(4 + \frac{x^3 - 64x}{100}\right)^2 dx \quad (1)$$

$$= \frac{\pi}{100^2} \int_{-8}^{8} (400 + x^3 - 64x)^2 \, dx$$

$$= \frac{\pi}{100^2} \int_{-8}^{8} 400^2 + {x^3}^2 + 64^2x^2 + 2(400x^3 + 400(-64x) - 64x(x^3)) \, dx$$

$$= \frac{\pi}{100^2} \int_{-8}^{8} (160000 + x^6 + 4096x^2 + 800x^3 - 51200x - 128x^4) \, dx$$

$$= \pi \int_{-8}^{8} \left(\frac{x^6}{10000} - \frac{128x^4}{10000} + \frac{800x^3}{10000} + \frac{4096x^2}{10000} - \frac{51200x}{10000} + \frac{160000}{10000}\right) dx$$

$$= \pi \int_{-8}^{8} \left(\frac{x^6}{1000} - \frac{8x^4}{625} + \frac{2x^3}{25} + \frac{256x^2}{625} - \frac{128x}{25} + 16\right) dx \quad (1)$$

DO NOT WRITE IN THIS AREA

Question 8 continued

$$= \pi \left[\frac{x^7}{70000} - \frac{8x^5}{3125} + \frac{x^4}{50} + \frac{256x^3}{1875} - \frac{64x^2}{25} + 16x \right]_{-8}^{8} \quad (1)$$

$$= \pi \left[\underbrace{(62.058..)}_{x=8} - \underbrace{(-225.898.)}_{x=-8} \right] \quad (1)$$

$$= 905 \text{ cm}^3 \quad (3.s.f) \quad (1)$$

(e) $905\text{cm}^3 = 905\text{ml}$. This is close to $1000 - 100 = 900\text{ml}$, so the model is suitable (1)

Question 8 continued

(Total for Question 8 is 15 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS